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Nonlinear dynamic response of floating piles under vertical vibration

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ABSTRACT: This paper presents the influence of non-linearity on the dynamic response of floating pile foundation subject to vertical vibration. A detailed theoretical investigation of stiffness and damping parameters is made for floating piles with different shear modulus reduction ratio of the weak zone soil. The accuracy of the non-linear analysis for predicting the dynamic response depends upon the choice of boundary zone parameters and the pile separation length. The tip resistance of pile is neglected and comparison of the theoretical curves with the experimental results has been made. In this study it is shown that close agreement between theoretical and measured response curves of floating piles can be achieved by considering precise boundary zone parameter values and separation between pile and soil.

1 INTRODUCTION

Floating Piles are Friction piles which transfer their load to ground through skin friction. Most often due to the improper cleaning of boreholes some amount of bentonite slurry (polymud) remains present at the bottom of the boreholes even after the installation of bored cast in situ concrete piles. Due to the improper contact between pile tip and soil, the end bearing resistance is not fully developed which is a common phenomenon in clayey soils. This results in higher amplitude of vibration under dynamic loading which can cause massive destruction to the machine foundations. Therefore there is a need to establish a method to calculate the dynamic nonlinear response of pile foundations under induced forces of rotation machines without considering the end bearing resistance of pile foundation.

The interaction between the pile and soil determines the load resistance and serviceability of the structure. This interaction results in development of stiffness and damping of the pile which can be expressed by impedance functions (Novak 1974). The total stiffness of the pile is complex where the real (in phase) part describes the true stiffness and complex (out of phase) part signifies total damping. Various approximate linear methods have been proposed to model the non-linear behavior of pile in which finite element approach offers accuracy with great versatility. Matlock et al. (1978) introduced lumped mass models with nonlinear discrete springs, dashpot, and friction elements to simulate the nonlinear behaviour of pile foundation. Novak and Grigg (1978) performed dynamic experiments and investigated frequency response of piles. Novak et al. (1978) proposed a linear soil model with a constant shear modulus to predict the dynamic behavior of piles. Later weak cylindrical zone around the pile with step variation of shear modulus was proposed by Novak and Sheta (1980) to get the nonlinear effect of pile soil system. Han (1997) introduced a parabolic variation of shear modulus for the weak inner zone of the pile to simulate a better of pile estimation of nonlinear pile soil response.

The main objective of the present study is to monitor the nonlinear dynamic response of floating piles under vertical vibration. To fulfill this objective the continuum approach analysis described by Novak and Aboul Ella (1978) has been used to model the response of single floating piles under vertical vibration of rotating machines ignoring the end bearing resistance of pile. To incorporate the nonlinear effect the soil model describe by Novak and Sheta (1980) assuming a cylindrical zone around the pile with less inner zone soil modulus than outer zone. The variation of dimensionless stiffness parameter f_{w1} and damping parameter f_{w2} with dimensionless frequency in floating piles with varying shear modulus reduction ratio has been investigated and presented. The effectiveness of the model is also monitored by comparing the theoretical response curve with experimental results (Manna and Baidya, 2010) of two different soil-pile separation conditions ignoring the end bearing resistance of the pile.

2 THEORETICAL STUDY

Among all the methods described by many researchers for predicting the response of pile foundations under dynamic load, the continuum approach is most promising and widely used. In Continuum Approach the stiffness and damping are calculated by solving the equation of motion which is derived by dividing the whole pile into two nodded one dimensional pile elements. This method is first presented by Novak and Aboul Ella (1978). Based on their assumptions and the model, a computer code SPVVA (Single – Pile Vertical Vibration Analyzer) has been developed in Matlab (R2012b) to calculate the nonlinear response of single piles without considering the pile tip reaction.

2.1 *Soil stiffness model*

The pile foundation resists the load by developing the soil-pile interaction phenomenon between the surrounding soils and the pile elements. The method of determine the dynamic soil stiffness has been described by many researchers. Here the general complex soil stiffness is derived for infinite long rigid embedded piles under uniform harmonic motions in vertical direction. The soil stiffness in vertical direction can be defined by

$$K_{wv} = G[S_{w1}(a_0, D) + iS_{w2}(a_0, D)] \quad (1)$$

where, dimensionless frequency $a_0 = r_0\omega/V_0$; $r_0 =$ equivalent radius of cross-section of pile; $V_0 =$ shear velocity; $\omega =$ frequency; $i = (-1)^{0.5}$; S_{w1} are S_{w2} real and imaginary parts of the dimensionless complex soil stiffness.

The magnitude and variation of soil properties along depth and radial direction plays a significant role in determining the dynamic soil stiffness and damping. These factors along with slippage, lack of bond are accounted for the inclusion of cylindrical zones around the pile whose shear modulus and material damping differ from those of the surround medium. Many models have been developed in an attempt to simulate the nonlinear soil behavior.

In this study the soil model describe by Novak and Sheta (1980) is used to approximate the soil nonlinear behavior. The soil is assumed to be composed of horizontal layers that are homogenous, isotropic, linear viscoelastic with frequency independent material damping. The pile is taken as rigid, circular, mass less and infinitely long cylinder with the boundary between the two media. A cylindrical annulus of mass less ($\rho = 0$) softer soil (an inner weakened zone) is considered around the pile than the outer medium. The soil stiffness and damping depends on inner weak zone damping (D_{ws}); shear modulus reduction ratio (G_{ws}/G_s); dimensionless frequency parameter (a_0) and thickness ratio (T_{ws}/D_p) and is shown in Figure 1. Weakened bond and slippage are accounted by reduced shear modulus and increased damping in the inner layer. Hence the vertical soil stiffness can be expressed as

$$K_{wv} = G_s\left(S_{w1}\left(a_0, \frac{G_{ws}}{G_s}, D_{ws}, \frac{T_{ws}}{D_p}, D_s\right) + iS_{w2}\left(a_0, \frac{G_{ws}}{G_s}, D_{ws}, \frac{T_{ws}}{D_p}, D_s\right)\right) \quad (2)$$

The variation of the dimensionless soil stiffness and damping parameters with dimensionless frequency for

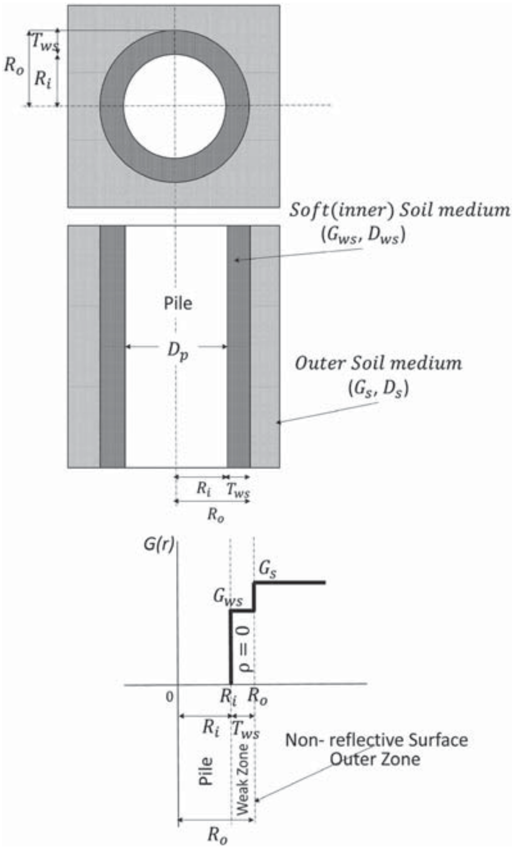

Figure 1. Schematic representation of cylindrical boundary zone around the pile and the variation of shear modulus of soil in the layered soil media.

different shear modulus reduction ratio and assuming other parameter constant is shown in Figure 2. It is observed from the curves that the value of S_{w1} and S_{w2} increases with the increase of shear modulus reduction ratio and frequency. At higher range of shear modulus reduction ratio the dimensionless soil stiffness and damping parameters are not increasing proportionally.

2.2 *Analytical model*

The impedance function of the pile in the composite medium is derived from the combination of element stiffness matrixes. The element stiffness matrix is derived for each pile element by considering homogeneous, vertical prismatic elements extending between the interfaces of each layer (Figure 3). The properties of each element are fully described by its complex stiffness matrix which includes the properties of both pile and soil. The embedded element reactions can be described by the differential equation of motion in the vertical direction $w(z,t)$ as

$$\mu\ddot{w} + c\dot{w} - E_p A \frac{d^2 w}{d z^2} + G(S_{w1} + iS_{w2})w = 0 \quad (3)$$

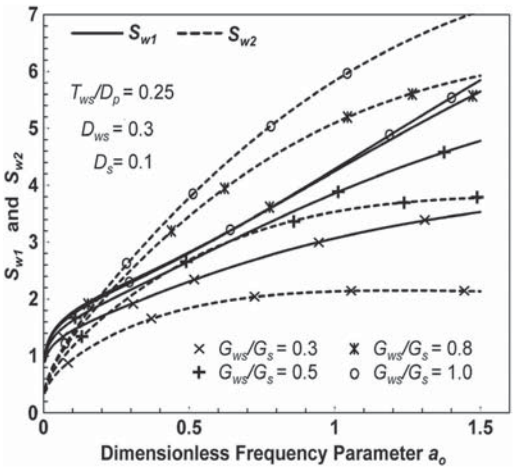

Figure 2. The variation of dimensionless soil stiffness and damping parameters with dimensionless frequency for different shear modulus reduction ratio.

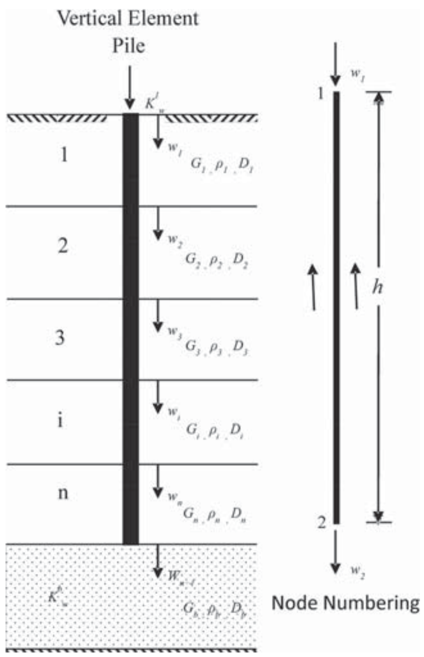

Figure 3. Pile embedded in soil strata with nodes numbering.

where, μ = mass of the pile per unit length; $\dot{w}$ = time derivative; c = coefficient of pile internal damping; E_p = Young's modulus and A = the cross-sectional area of the pile respectively.

Assuming the harmonic motion $w(z,t) = w(z,t)e^{-i\omega t}$ having complex amplitude $w(z)$, the above equation can be simplified to

$$\frac{d^2 w}{d z^2} + \left(\frac{\lambda}{h}\right) 2z = 0 \tag{4}$$

where, the value λ is the equivalent complex frequency parameter. Thus the complex amplitude can be written as the following general equation in which B, C = integration constants

$$w(z) = B\cos\left(\frac{\lambda}{h}\right) + C\sin\left(\frac{\lambda}{h}\right) \tag{5}$$

$$\lambda = h\sqrt{\frac{\left(\mu\omega^2 - G S_{w1} - i\left(c\omega + G S_{w2}\right)\right)}{E_p A}} \tag{6}$$

2.2.1 *Dynamic stiffness matrix*

The dynamic stiffness of the pile can be defined by $w(0) = 1$, $w(h) = 0$ and $w(0) = 0$, $w(h) = 1$ boundary conditions. Let consider a general element just beneath the soil layer as shown in the Figure 1 of length h. In the figure, the nodes have been numbered as i and $i+1$ down the depth of the element i. The amplitude of the axial force $N(z)$ can be represented by the following equation up to the elastic range of pile material.

$$N(z) = -E_p A \frac{dw(z)}{dz} \tag{7}$$

Considering first as $w(0) = 1$ and $w(h) = 0$ i.e. unit displacement at the node i with other node $i+1$ fixed give the value of $B = 1$ and $C = -\cot(\lambda)$.

$$K_{i,i} = N(0) = \frac{E_p A}{h}\lambda\cot(\lambda) \tag{8}$$

$$K_{i,i+1} = N(h) = -\frac{E_p A}{h}\lambda\operatorname{cosec}(\lambda) \tag{9}$$

Similarly when $w(0) = 0$ and $w(h) = 1$ i.e. unit displacement at the node $i+1$ with other node i fixed give the value of $B = 0$ and $C = \operatorname{cosec}(\lambda)$

$$K_{i+1,i+1} = N(0) = \frac{E_p A}{h}\lambda\cot(\lambda) \tag{10}$$

$$K_{i+1,i} = N(h) = -\frac{E_p A}{h}\lambda\operatorname{cosec}(\lambda) \tag{11}$$

Therefore the overall stiffness of the individual member can be formulated in the matrix form as

$$\left[K_w\right] = \frac{E_p A}{h}\lambda\begin{bmatrix}\cot(\lambda) & -\operatorname{cosec}(\lambda)\\ -\operatorname{cosec}(\lambda) & \cot(\lambda)\end{bmatrix} \tag{12}$$

Then the end forces N_1 and N_2 corresponding to end displacements w_1 and w_2 are expressed as

$$\begin{Bmatrix}N_1\\ N_2\end{Bmatrix} = \left[K_w\right]\begin{bmatrix}w_1\\ w_2\end{bmatrix} \tag{13}$$

The total stiffness of the pile can be integrated from the elemental matrix diagonally. The bandwidth of the stiffness matrix is 2.

2.2.2 *Vertical stiffness of the pile*

For the vertical direction, the stiffness matrix [K_w] is assembled and using this stiffness matrix, the complex vertical stiffness of the pile at different nodes can be derived by

$$\begin{Bmatrix} K_w^1 \\ K_w^2 \\ \vdots \\ \vdots \\ K_w^{n+1} \end{Bmatrix} = \begin{bmatrix} K_w \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ \vdots \\ w_{n+1} \end{bmatrix} \tag{14}$$

where, K_w^n is the corresponding force at node n and w_n is the displacement of node n and [K_w] can be represented by a band matrix of 2 as

$$K_w = \begin{bmatrix} K_{11} & K_{12} & & & & \\ K_{21} & K_{22}+K_{22} & K_{23} & & & \\ & K_{32} & K_{33}+k_{33} & & & \\ & & & \ddots & & \\ & & & \ddots & K_{n-1n} & \\ & & & & K_{nn}+K_{nn} & K_{nn+1} \\ & & & & K_{n+1n} & K_{n+1n+1} \end{bmatrix} \tag{15}$$

The settlement of any structure can be found directly by the settlement of pile head, so it is of interest to find the stiffness of the pile at tip. Hence the stiffness K_w^1 is derived as described below

$$\begin{Bmatrix} K_w^1 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{Bmatrix} = \begin{bmatrix} K_w \end{bmatrix} \begin{bmatrix} 1 \\ w_2 \\ \vdots \\ \vdots \\ w_{n+1} \end{bmatrix} \tag{16}$$

$$K_w^1 = 1 \Big/ \left[\left[K_w \right]^{-1} \right]_{1,1} \tag{17}$$

The stiffness of the pile head is complex where, the real part corresponds to the true stiffness k_w^1 and imaginary part corresponds to the equivalent viscous damping C_w^1. Thus the complex vertical stiffness of single pile is expressed as

$$K_w^1 = k_w^1 + i\omega C_w^1 \tag{18}$$

The dimensionless stiffness and damping parameters f_{w1} and f_{w2} can be calculated as

$$f_{w1} = real\left(K_w^1\right)\frac{r}{E_p A} \tag{19}$$

$$f_{w2} = img\left(K_w^1\right)\frac{V_s/\omega}{E_p A} \tag{20}$$

where r, E_p, A are of the top-most layer and V_s is of the bottom most layer.

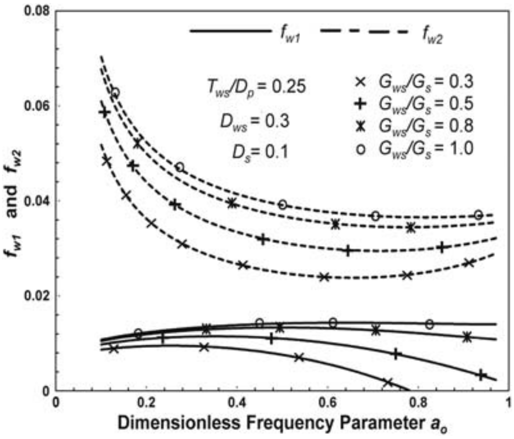

Figure 4. The variation of dimensionless stiffness and damping parameters with dimensionless frequency for different shear modulus reduction ratio.

3 STIFFNESS AND DAMPING PARAMETERS

The behavior of the pile in vertical vibration can be characterized by the dimensionless Stiffness parameter f_{w1} and damping parameter f_{w2}. These parameters depends upon the boundary zone parameters like shear modulus reduction ratio (G_{ws}/G_s), inner weak zone damping (D_{ws}), thickness ratio (T_{ws}/D_p) and also on separation length at pile soil interface (S_L).

In this present study a detailed comparison of dimensionless stiffness and damping parameters (f_{w1} and f_{w2}) are presented for different values of shear modulus reduction ratio (G_{ws}/G_s) = 0.3, 0.5, 0.8 and 1. Other boundary zone parameters are kept constant which are assumed to be as 0.25 for thickness ratio; 0.1 for outer soil damping and 0.3 for inner weak zone damping. The length of the pile is taken as 2 m which is divided into 10 elements.

The variation of f_{w1} and f_{w2} with dimensionless frequency for different shear modulus reduction ratio under vertical vibration is presented in Figure 4. It is observed from the figure that the dimensionless stiffness and damping parameters increases with the increase of shear modulus reduction ratio. However, the increment is not very significant for higher range of shear modulus reduction ratio values. It is also noted that at lower frequency range, the value of damping parameter is decreasing rapidly because the viscous damping of the system is inversely proportional with frequency. Whereas the stiffness parameter is almost remain constant with frequency.

4 COMPARATIVE STUDY

Based on Novak's continuum approach (Novak and Aboul-Ella, 1978) SPVVA1 (Single – Pile Vertical Vibration Analyzer Program) is developed in Matlab

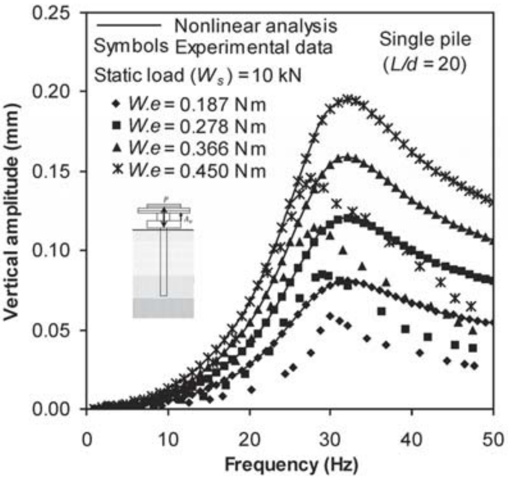

Figure 5. Comparison of experimental and theoretical response of single pile without pile-soil separation.

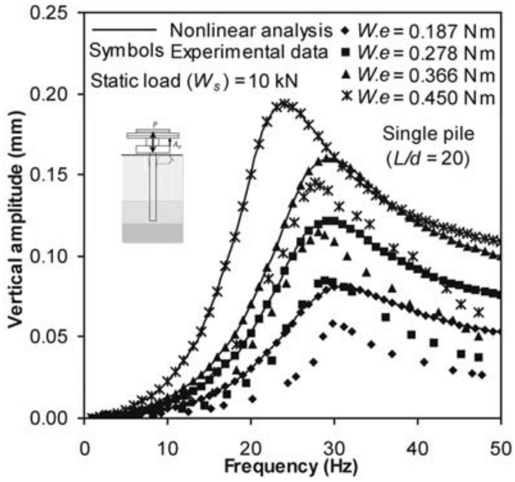

Figure 6. Comparison of experimental and theoretical response of single pile considering pile-soil separation.

(R2012b) to determine the impedance function of the single pile subject to vertical vibration.

The results obtained from this compute program are compared with the experimental results which are reported by Manna and Baidya (2010) using same soil as well as boundary zone properties presented in that paper. In that study the forced vibration tests of single piles were conducted at a site located adjacent to Hangar, at Indian Institute of Technology, Kharagpur Campus, India. The soil consisted of a 1.2 m layered stratum of yellow organic silty clay with low plasticity, resting on a thick 1.1 m stratum of brown medium stiff inorganic clay which itself rested on gravel mixed-stiff inorganic clay at a depth of 2.3 m. The variation of shear wave velocity V_s of soil layers was obtained using established empirical equations.

Here, the frequency-amplitude response single pile obtained from the developed non-linear computer program is compared with the experimental results presented in Manna and Baidya (2010). A single pile results of $l/d = 20$ and static load $(W_s) = 10$ kN are used for comparison with and without introducing the length of separation between pile and soil from the ground level (S_L). The pile tip resistance is assumed to be zero to stimulate the floating piles condition.

As previously discussed two soil-pile systems are modeled here, case I – without considering pile-soil separation length and case II – with consideration of pile-soil separation length (S_L). In total four eccentricities ($W.e = 0.187$, 0.278, 0.366 and 0.450 Nm, where $W =$ weight of two counter rotating masses of the oscillator and $e =$ eccentric distance between the masses) are considered for both the cases and the separation lengths ($S_L = 0.18D_p$ for $W.e = 0.187$ Nm to $2.4D_p$ for $W.e = 0.450$ Nm) are taken as described in the Manna and Baidya (2010) for pile with diameter (D_p) of 0.1 m. The nonlinear dynamic responses are calculated by introducing a cylindrical weak zone around the pile and the nonlinear boundary zone parameters are also considered according to that study.

The comparison curves considering the without and with separation condition are shown in Figure 5 and Figure 6 respectively. It can be noted from both the comparison curves that the theoretical curves are quite well match with the experimental results though in higher eccentric moment it differs a little. The nonlinearity is also observed with the phenomenon of decreasing resonant frequencies with the increasing excitation intensity and also the amplitudes are not proportional to the excitation intensity. For the modeled graphs with and without separation length resemble each other except at higher excitation intensities where increase in pile separation decreases stiffness of the pile foundation which results the decrease in the resonant frequency of the system. The little discrepancy in amplitude is may be due to the development of end bearing in model pile but not considered in the case of analysis.

5 CONCLUSIONS

The principle objective of the present study is to investigate the non-linear frequency amplitude response of floating piles under vertical vibration and to determine the variation of stiffness and damping parameters f_{w1} and f_{w2} with frequency. A comparative study between the theoretical and experimental response curves considering with and without pile separation is also been presented. The findings of this study have provided a clear insight about the soil-pile interaction phenomenon on dynamic responses of floating piles under vertical vibration. Some major conclusions that can be made from the theoretical study are summarized below.

1. The soil-pile interaction has a significant effect on the dynamics of pile which is itself largely

dependent upon the boundary zone parameters and pile separation length. However the introduction of separation length leads to an overall reduction in pile-soil system stiffness.
2. The dimensionless stiffness and damping parameters f_{w1} and f_{w2} also depend upon the boundary zone parameters and soil-pile separation length. In this study the predicted dimensionless stiffness and damping parameters follow a very evident trend under vertical vibration.
3. It is found from the frequency-amplitude comparison curves that the theoretical curves are quite well match with the experimental results. The resonant frequencies of both theoretical and experimental curves are almost matches whereas the amplitude results are little bit higher because of the assumption of lower damping value of soil for the analysis. This is may be due to the development of end bearing in model pile which has not been considered for analysis.

Finally it can be concluded that the present model can predict the dynamic nonlinear response of single floating piles reasonably well with the accurate assumption of nonlinear boundary zone parameters and separation length. Though considering other transition function between week soil zone and outer soil layer may produce more realistic results for predicting the nonlinear response of pile foundation.

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