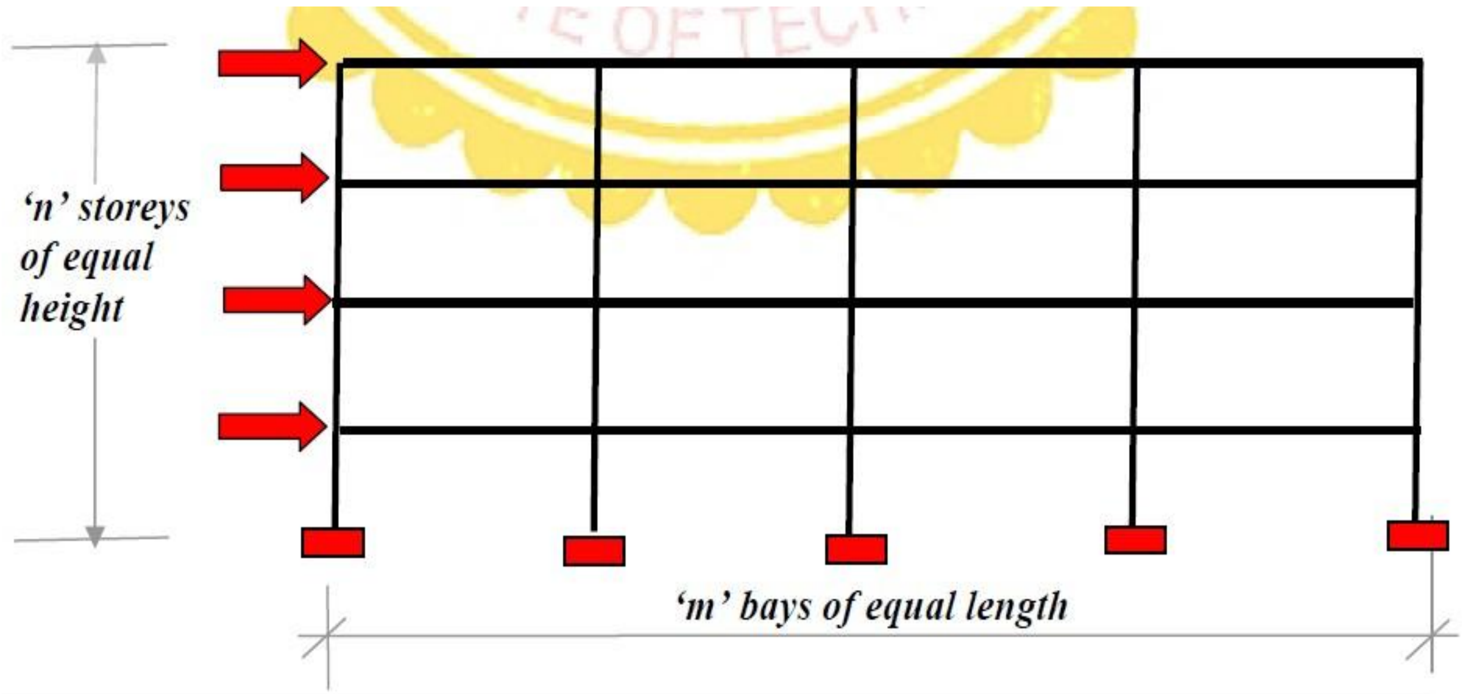
# Project Report
# Structure Assignment B





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# The Direct Stiffness Method: Plane Frames

## Introduction

In the case of plane frame, all the members lie in the same plane and are interconnected by rigid joints. The internal stress resultants at a cross-section of a plane frame member consist of bending moment, shear force and an axial force. The significant deformations in the plane frame are only flexural and axial. In this lesson, the analysis of plane frame by direct stiffness matrix method is discussed. Initially, the stiffness matrix of the plane frame member is derived in its local co-ordinate axes and then it is transformed to global co-ordinate system. In the case of plane frames, members are oriented in different directions and hence before forming the global stiffness matrix it is necessary to refer all the member stiffness matrices to the same set of axes. This is achieved by transformation of forces and displacements to global co-ordinate system.

## Member Stiffness Matrix

Consider a member of a plane frame as shown in Fig. 30.1a in the member coordinate system $x' \, y' \, z'$. The global orthogonal set of axes $xyz$ is also shown in the figure. The frame lies in the $xy$ plane. The member is assumed to have uniform flexural rigidity $EI$ and uniform axial rigidity $EA$ for sake of simplicity. The axial deformation of member will be considered in the analysis. The possible displacements at each node of the member are: translation in $x'$ - and $y'$ - direction and rotation about $z'$- axis.

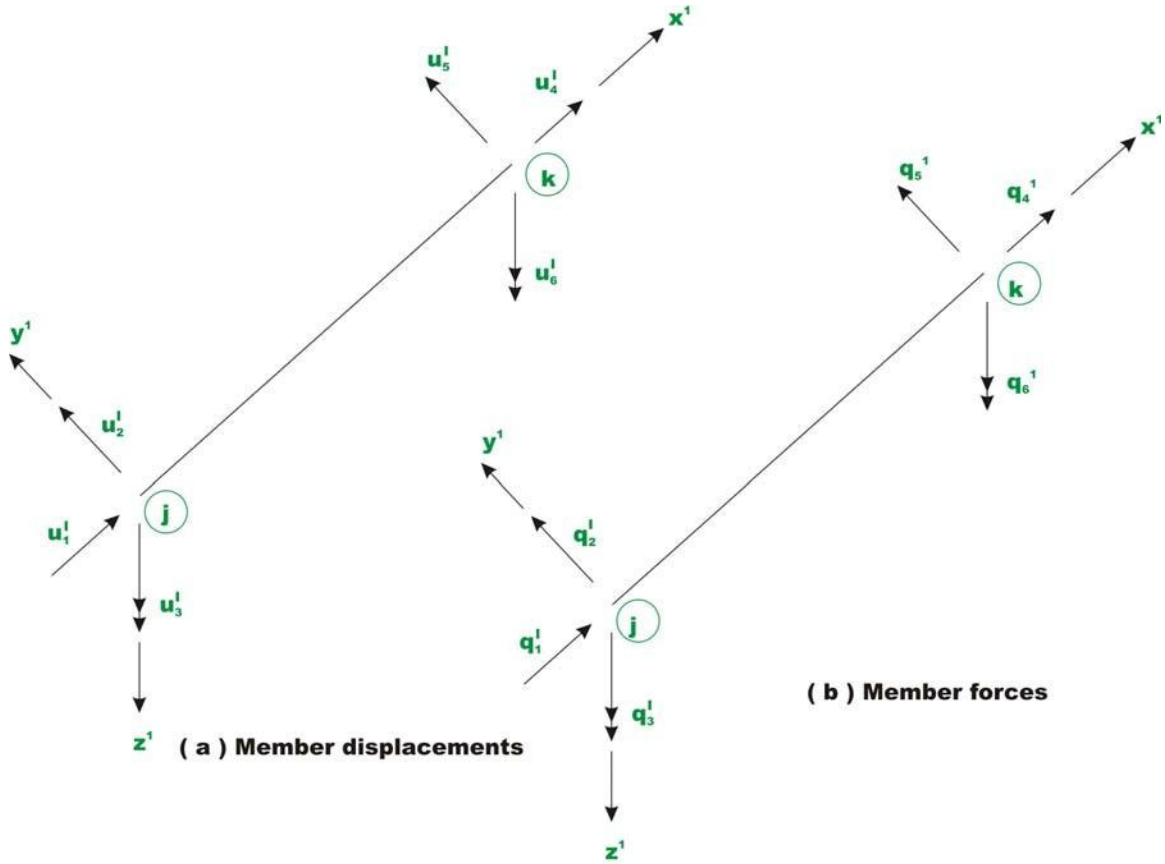


**Fig. 30.1 Frame member in local co-ordinate system**

Thus the frame members have six (6) degrees of freedom and are shown in Fig.30.1a. The forces acting on the member at end $j$ and $k$ are shown in Fig. 30.1b. The relation between axial displacement and axial forces is derived in chapter 24. Similarly the relation between shear force, bending moment with translation along $y'$ axis and rotation about $z'$ axis are given in lesson 27. Combining them, we could write the load-displacement relation in the local coordinate axes for the plane frame as shown in *Fig 30.1a, b* as,

This may be written as

$$\{q'\} = [k']\{u'\} \qquad (30.1b)$$

where $[k']$ is the member stiffness matrix. The member stiffness matrix can also be generated by giving unit displacement along each possible displacement degree of freedom one at a time and calculating resulting restraint actions.

## Transformation from local to global co-ordinate system

### *Displacement transformation matrix*

In plane frame the members are oriented in different directions and hence it is necessary to transform stiffness matrix of individual members from local to global co-ordinate system before formulating the global stiffness matrix by assembly. In Fig. 30.2a the plane frame member is shown in local coordinate axes $x'y'z'$ and in Fig. 30.2b, the plane frame is shown in global coordinate axes $xyz$ . Two ends of the plane frame member are identified by $j$ and$k$ . Let $u'_1, u'_2, u'_3$ and $u'_4, u'_5, u'_6$ be respectively displacements of ends $j$ and $k$ of the member in local coordinate system $x'\ y'\ z'$. Similarly $u_1, u_2, u_3$ and $u_4, u_5, u_6$ respectively are displacements of ends $j$ and $k$ of the member in global co-ordinate system.

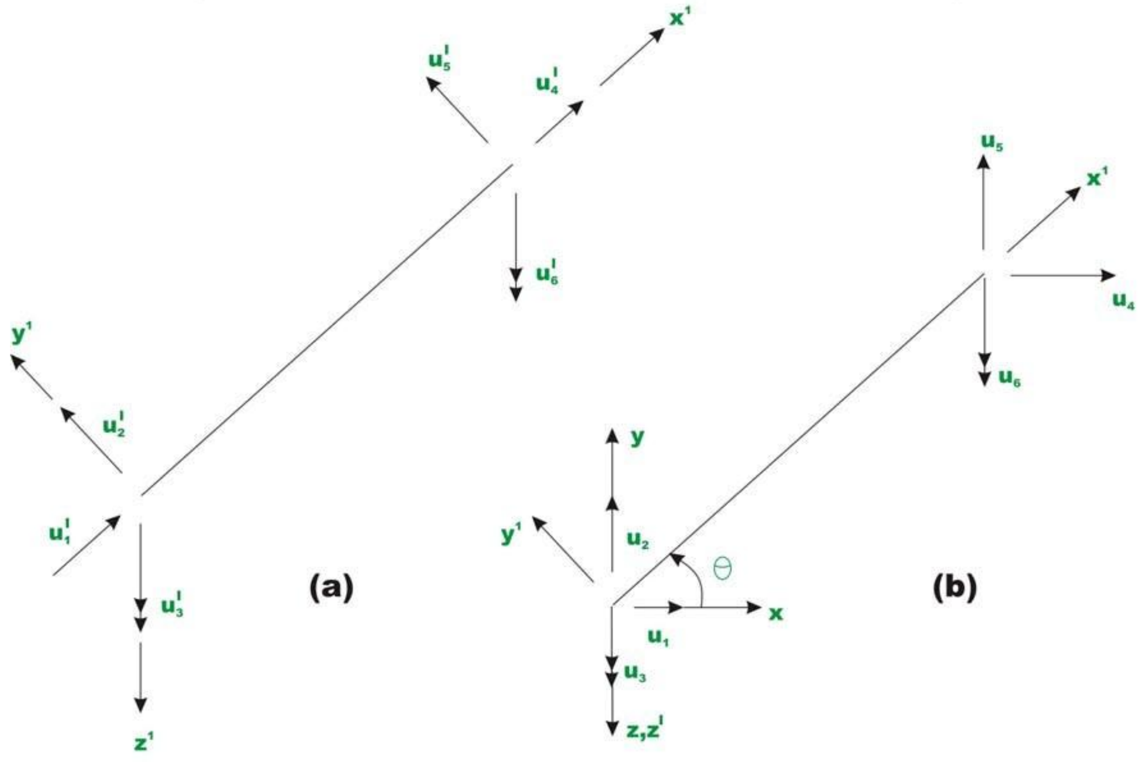


**Fig. 30.2 Plane frame member in**
**(a) Local co-ordinate system**
**(b) Global co-ordinate system.**

Let θ be the angle by which the member is inclined to global $x$-axis. From Fig.30.2a and b, one could relate $u'_1, u'_2, u'_3$ to $u_1, u_2, u_3$ as,

$$u'_1 = u_1 \cos\theta + u_2 \sin\theta \qquad (30.2a)$$

$$u'_2 = -u_1 \sin\theta + u_2 \cos\theta \qquad (30.2b)$$

$$u'_3 = u_3 \qquad (30.2c)$$

This may be written as,

$$\begin{Bmatrix} u'_1 \\ u'_2 \\ u'_3 \\ u'_4 \\ u'_5 \\ u'_6 \end{Bmatrix} = \left[\begin{array}{ccc|ccc} l & m & 0 & 0 & 0 & 0 \\ -m & l & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & l & m & 0 \\ 0 & 0 & 0 & -m & l & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} \qquad (30.3a)$$

Where, $l = \cos\theta$ and $m = \sin\theta$.

This may be written in compact form as,

$$\{u'\} = [T]\{u\} \qquad (30.3b)$$

In the above equation, $[T]$ is defined as the displacement transformation matrix and it transforms the six global displacement components to six displacement components in local co-ordinate axes. Again, if the coordinate of node $j$ is

$(x_1, y_1)$ and coordinate of node $k$ are $(x_2, y_2)$, then,

***Force displacement matrix***

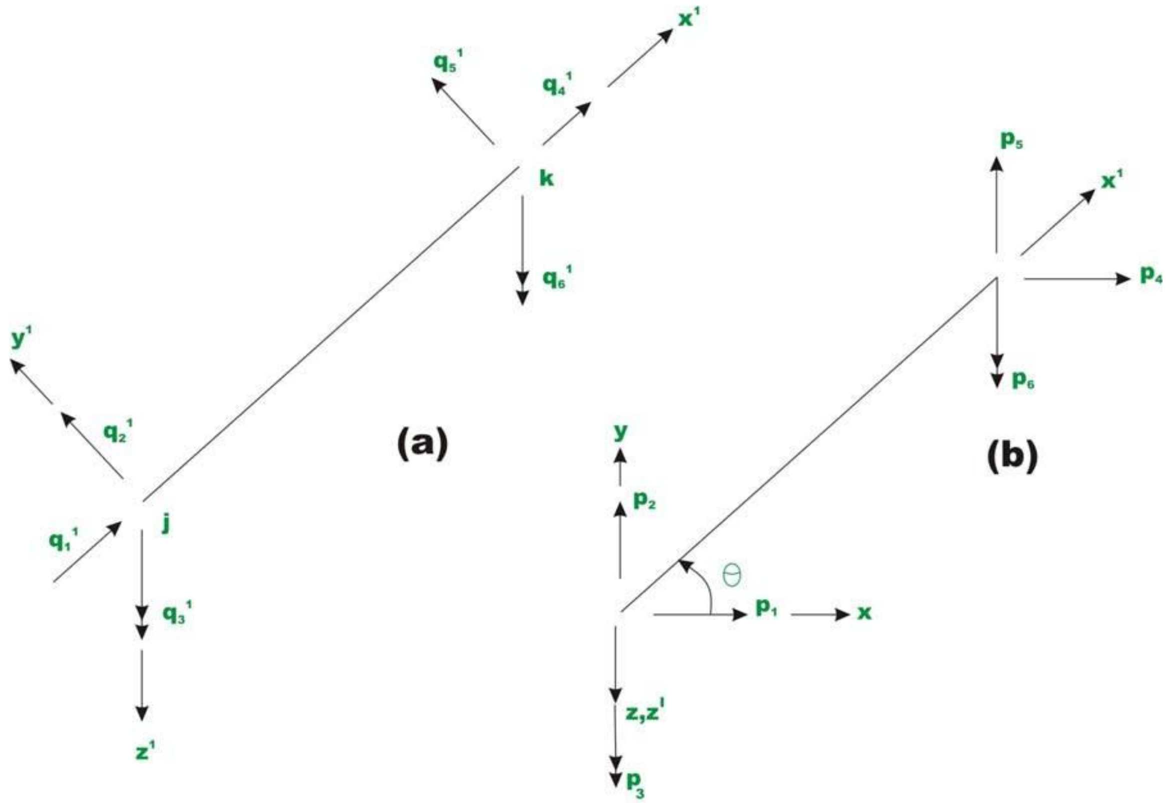


**Fig. 30.3 Plane frame member in**
**(a) Local co-ordinate axes and**
**(b) In global co-ordinate system**

Let $q'_1, q'_2, q'_3$ and $q'_4, q'_5, q'_6$ be respectively the forces in member at nodes $j$ and $k$ as shown in Fig. 30.3a in local coordinate system. $p_1$, $p_2$, $p_3$ and $p_4$, $p_5$, $p_6$ are the forces in members at node $j$ and $k$ respectively as shown in Fig. 30.3b in the global coordinate system. Now from Fig 30.3a and b,

$$p_1 = q'_1 \cos\theta - q'_2 \sin\theta \tag{30.5a}$$

$$p_2 = q'_1 \sin\theta + q'_2 \cos\theta \tag{30.5b}$$

$$p_3 = q'_3 \tag{30.5c}$$

Thus the forces in global coordinate system can be related to forces in local coordinate system by

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{Bmatrix} = \left[\begin{array}{ccc|ccc} l & -m & 0 & 0 & 0 & 0 \\ m & l & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & l & -m & 0 \\ 0 & 0 & 0 & m & l & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} q'_1 \\ q'_2 \\ q'_3 \\ q'_4 \\ q'_5 \\ q'_6 \end{Bmatrix} \tag{30.6a}$$

Where, $l = \cos\theta$ and $m = \sin\theta$.

This may be compactly written as,

$$\{p\} = [T]^T \{q'\} \tag{30.6b}$$

***Member global stiffness matrix***

From equation (30.1b), we have

$$\{q'\} = [k']\{u'\}$$

Substituting the above value of $\{q'\}$ in equation (30.6b) results in,

$$\{p\} = [T]^T [k']\{u'\} \tag{30.7}$$

Making use of equation (30.3b), the above equation may be written as

$$\{p\} = [T]^T [k'][T]\{u\} \tag{30.8}$$

or

$$\{p\} = [k]\{u\} \tag{30.9}$$

The equation (30.9) represents the member load-displacement relation in global coordinate system. The global member stiffness matrix $[k]$ is given by,

$$[k] = [T]^T [k'] [T] \qquad (30.10)$$

After transformation, the assembly of member stiffness matrices is carried out in a similar procedure as discussed for truss. Finally the global load-displacement equation is written as in the case of continuous beam. Few numerical problems are solved by direct stiffness method to illustrate the procedure discussed.

## Summary

In this lesson, the analysis of plane frame by the direct stiffness matrix method is discussed. Initially, the stiffness matrix of the plane frame member is derived in its local co-ordinate axes and then it is transformed to global co-ordinate system. In the case of plane frames, members are oriented in different directions and hence before forming the global stiffness matrix it is necessary to refer all the member stiffness matrices to the same set of axes. This is achieved by transformation of forces and displacements to global co-ordinate system. In the end, a few problems are solved to illustrate the methodology.

**Source: Nptel and other website**

# Our Procedure:

The major steps in solving any planar frame problem using the direct stiffness method:

***Step 1***: Select the problem units. Set up the coordinate system. Identify and label the nodes and the elements based on the values of m and n. If the number of bays is greater than the number of storeys, numbering of nodes is done vertically, else otherwise. For each element select a start node (node 1) and an end node (node 2). Label the three global dof at each node starting at node 1 and proceeding sequentially.

***Step 2***: Assign each member its local and global stiffness matrix using the value of the angle it makes with the horizontal.

***Step 3***: If there are element loads, compute the equivalent joint loads and transform them to the global coordinate system. Note that if there is more than one element load acting on an element, use linear superposition (algebraic sum) of all the element loads acting on that element. In our case, we have only elemental loads at the nodes, thus superposition is not required.

***Step 4***: Form stiffness matrix of the total structure using each members' global stiffness matrix.

***Step 5***: Solve the system equations **KD = F** for the nodal displacements **D**.

***Step 6***: For each element using the nodal displacements, compute the element nodal forces and displacements.